

Gluon condensation at finite temperature via AdS/CFT

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ABSTRACT: We consider gluon condensation (GC) at finite temperature using AdS/CFT. We first show that in the presence of regular horizon, the GC is forbidden in high temperature. Then we consider gravity back-reaction to dilaton coupling and show that the back-reaction develops a singularity, and non-vanishing value of gluon condensation is allowed. We also study thermodynamic quantities and the trace anomaly in the presence of the GC. We discuss how to define a temperature in the presence of the singularity which forbids Hawking temperature. Finally we describe the thermodynamics of the gluon condensation including the effect of the Hawking-Page transition.

KEYWORDS: Gauge-gravity correspondence, QCD, Black Holes in String Theory.

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1. Introduction

There are increasing activities in using AdS/CFT [1] to describe the real systems after relevant deformations of the AdS background. For example, it has been suggested that the fireball in Relativistic Heavy Ion Collision (RHIC) should be considered as a strongly interacting system [2, 3] and the fireball has been studied using dual gravity models [4–11]. In the finite temperature context the SUSY is broken, and therefore the models have more chance to be in the same universality class of real QCD. Many attempts were made to construct models phenomenologically closer to QCD [12] as well as models with mesons and quenched quarks [13–16].

More recently, in an interesting paper [17], dilaton-gravity solution describing the gluon condensation at zero temperature [17] was discussed. We call this solution as dilaton-wall solution. In fact, dilaton-wall solution has a rather long history of being repeatedly rediscovered [18–20]. The gluon condensate was originally introduced, at zero temperature, by Shifman, Vainshtein and Zakharov (SVZ) as a measure for nonperturbative physics in QCD [21]. At high temperature, it is useful to study the nonperturbative nature of the quark-gluon plasma (QGP), and it can be served as an order parameter for (de)confinement [22–24]. Recently the role of the gluon condensate in RHIC physics is extensively studied in [25]. Here motivated by the recent paper [17], we describe the temperature dependence of the gluon condensation by extending dilaton-wall solution to the finite temperature. We consider the back-reaction of the ads black hole solution and the immediate consequences are of two folds: one is the development of the singularity (instead of regular horizon) and the other is the breaking of the conformal symmetry. We shows that the singularity plays a crucial role to allow the non-vanishing value of gluon condensation.

We also study thermodynamic quantities and the trace anomaly in the presence of the GC. Although the singularity of the solution does not admit the Hawking temperature, we argue that we can nevertheless define the temperature using the holographic correspondence of an extended Stephan-Boltzmann law. Finally we describe the temperature dependence of the gluon condensation including the effect of the Hawking-Page transition (HPT).

The rest of the paper goes as follows. In section 2, we first consider the hard wall model [29], and then we discuss the back-reaction due to the gluon condensation in section 3. Our solution interpolates the dilaton-wall solution and the well-known AdS black hole solution. Since the solution does not admit the Hawking temperature in generic parameter values, we define the temperature using the temperature in the absence of condensation and justify using the holographic correspondences. In section 4, we will discuss the thermodynamics of the gluon condensation by calculating various thermodynamic potentials. In section 5, we consider temperature dependence of the gluon condensation including Hawking-Page transition [26–28]. In the discussion section, we describe some limitations of the present approach and a few future works.

2. Gluon condensation in hard wall model

The gluon condensation is in fact the simplest quantity in AdS/CFT consideration, since it couples with the dilaton which is a massless scalar. Here we use a simple model, called hard wall model, where confinement is treated by introducing the IR cut off. The action is given by a scalar coupled with background gravity minimally.

$$S = -\frac{1}{2\kappa^2} \int d^4x dz \sqrt{g} \partial_\mu \phi \partial^\mu \phi. \tag{2.1}$$

The equation of motion for this massless field is

$$\partial_z(z^{-3} f(z) \partial_z \phi) = 0. \tag{2.2}$$

In low temperature, the thermal AdS background is dominating for which $f(z) = 1$, and the solution is given by

$$\phi(z) = c_0 + c_1 z^4. \tag{2.3}$$

If we have a hard wall at z_m and UV boundary at $z = 0$, the proper boundary should be the Dirichlet boundary condition (BC) at the wall and at the AdS boundary $z = 0$:

$$\phi(z_m) = A, \quad \phi(0) = B. \tag{2.4}$$

These boundary conditions determine c_i 's as $c_0 = B$, and $c_1 = (A - B)/z_m^4$. On the other hand, the general AdS/CFT dictionary identifies c_1 as the gluon condensation

$$c_1 \sim \langle \text{Tr} F_{\mu\nu}^2 \rangle. \tag{2.5}$$

The point here is that it is temperature independent in a confining phase, simply because there is no explicit temperature dependence in the background metric. In fact this is

consistent with the recent result in large N gauge theory result [33].¹ The temperature dependence is suppressed for chiral condensation as well as gluon condensation in the gauge theory context.

According to Herzog [28], there is a Hawking-Page type transition at the $T = 2^{1/4}/\pi z_m$, and beyond this temperature the AdS black hole background is dominant.

$$ds^2 = \frac{R^2}{z^2} \left[dz^2 + (1 + az^4) \left(d\vec{x}^2 - \left(\frac{1 - az^4}{1 + az^4} \right)^2 dt^2 \right) \right]. \quad (2.6)$$

Here a is related to the temperature by $a = (\pi T)^4/4$. The scalar equation can be solved with $f(z) = 1 - a^2 z^8$. The solution is

$$\phi(z) = \phi_0 + \sqrt{\frac{3}{2}} \frac{c}{a} \log \frac{1 + az^4}{1 - az^4}, \quad (2.7)$$

While the UV boundary condition can be easily specified, we can not specify any regular boundary condition at the horizon unless $c = 0$. If that is the case, then such regularity requirement says that gluon condensation is 0 in deconfined phase.

To understand the regularity requirement at the boundary and also for the later comparison, we calculate the Lagrangian:

$$\mathcal{L} = \mathcal{R} + \frac{12}{R^2} - \frac{1}{2} \partial_M \phi \partial^M \phi. \quad (2.8)$$

The matter part is

$$-\frac{1}{2} g^{zz} (\partial_z \phi)^2 = -\frac{48c^2 z^8}{(1 - a^2 z^8)^2}, \quad (2.9)$$

which is singular at the IR boundary $z = z_T = a^{-1/4}$. The gravity part is free from IR singularity:

$$\mathcal{R} + \frac{12}{R^2} = -8. \quad (2.10)$$

Therefore if c is non-zero, there is no way to cancel out the IR divergence, confirming above requirement.

Therefore hard wall model prediction for the gluon condensation is the jump from a constant finite value to 0 at the critical temperature. See the figure 1b. The result is good qualitatively but *NOT very satisfactory*, since the lattice data shows that gluon condensation is non-zero at high temperature.

As we will see in the next section, including the gravity back-reaction changes the situation.

3. Metric back-reaction to the gluon condensation

Now we consider the back-reaction of the metric to the gluon condensation. We are mainly interested in the high temperature phase behavior of the gluon condensation. The 5D

¹The temperature independence of the chiral condensation were reported long ago [30, 31], and in the AdS/CFT context we argued in the same line here [32].

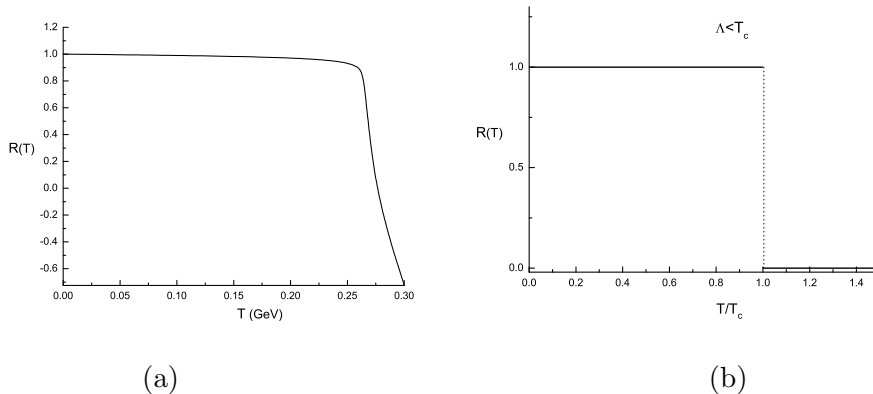


Figure 1: (a) Lattice result of Miller [24] for pure Yang Mills case, (b) The gluon condensate at finite temperature in Hard wall model.

gravity action with a dilaton is given by

$$S = \gamma \frac{1}{2\kappa^2} \int d^5x \sqrt{g} \left[\mathcal{R} + \frac{12}{R^2} - \frac{1}{2} \partial_M \phi \partial^M \phi \right], \quad (3.1)$$

where $\gamma = +1$ for Minkowski metric, and $\gamma = -1$ for Euclidean signature. We work with Minkowski metric for most cases in this paper.

The supersymmetric solution of this system is discovered in last decade [19, 20] to discuss the running coupling and confinement and rediscovered recently in [17] to discuss the gluon condensation at zero temperature. We call it as the dilaton-wall solution and follow the notation of Csaki and Reese [17] closely, where the metric is given by

$$ds^2 = \left(\frac{R}{z} \right)^2 \left(\sqrt{1 - c^2 z^8} \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 \right), \quad (3.2)$$

and the corresponding dilaton profile is given by

$$\phi(z) = \sqrt{\frac{3}{2}} \log \left(\frac{1 + cz^4}{1 - cz^4} \right) + \phi_0, \quad (3.3)$$

where ϕ_0 is a constant.

Below we give a two-parameter non-BPS solution that contains both the dilaton-wall solution as well as AdS black hole solution as its limiting cases.² We start from an ansatz

$$\begin{aligned} ds^2 &= \frac{R^2}{z^2} dz^2 + e^{2A(z)} (d\vec{x}^2 + e^{2B(z)} dt^2), \\ \phi &= \phi(z). \end{aligned} \quad (3.4)$$

²After the first version of this paper was uploaded, we were informed that our solution also was found earlier in [34, 35] in other context. So we do not claim any originality in discovering this metric.

The action and the metric ansatz lead us to the equations of motion:

$$\begin{aligned}
 -z^2\phi'^2 &= 12z^2A'' + 12zA' + 24z^2A'^2 - 24 \\
 0 &= 4zB' + 4z^2B'^2 + 4z^2B'' + 16z^2A'B' \\
 z^2\phi'^2 &= 24z^2A'^2 + 12z^2A'B' - 24 \\
 0 &= z^2\phi'' + z\phi' + 4z^2A'\phi' + z^2B'\phi',
 \end{aligned}
 \tag{3.5}$$

where a prime represents a derivative with respect to z . We are looking for a solution which is asymptotically AdS and reduces to the AdS black hole in one limit and also leads to the above BPS solution in some other limit.

After some efforts, we find a solution that is given by

$$\begin{aligned}
 A(z) &= -\log \frac{z}{R} + \frac{a}{4f} \log \left(\frac{1+fz^4}{1-fz^4} \right) + \frac{1}{4} \log(1-f^2z^8) \\
 B(z) &= -\frac{a}{f} \log \left(\frac{1+fz^4}{1-fz^4} \right) \\
 \phi(z) &= \phi_0 + \frac{c}{f} \sqrt{\frac{3}{2}} \log \left(\frac{1+fz^4}{1-fz^4} \right).
 \end{aligned}
 \tag{3.6}$$

As a result, the dilaton black hole metric reads

$$ds^2 = \frac{R^2}{z^2} \left[dz^2 + (1-f^2z^8)^{1/2} \left(\frac{1+fz^4}{1-fz^4} \right)^{a/2f} \left(d\vec{x}^2 - \left(\frac{1-fz^4}{1+fz^4} \right)^{2a/f} dt^2 \right) \right].
 \tag{3.7}$$

Here, f , which determines the position of the singularity, is related to a and c by a Pythagorean relation:

$$f^2 = a^2 + c^2.
 \tag{3.8}$$

The parameters a and c , the z^4 coefficient of the metric and dilaton field respectively, determine the temperature and the gluon condensation respectively when the other is 0. Notice that the solution is well defined only in the range $0 < z < f^{-1/4} := z_f$. Since $1/z$ is the energy scale of the boundary theory in AdS/CFT, z_f can be considered as the IR cut-off. One should also notice that

- For $a = 0$, the solution reduces to dilaton-wall solution.
- For $c = 0$, it becomes the Schwarzschild black hole solution (2.6) [8]. In this case, the parameter a and the temperature is related by $a = \frac{1}{4}(\pi T)^4$.

Therefore we expect that our solution describes the finite temperature with the gluon condensation. However, for the generic value of f , the metric has an essential singularity at $z = f^{-1/4}$ and the Hawking temperature can NOT be determined by requiring the absence of conical singularity at the horizon. See the appendix. So we have to answer following question associated with the metric:

- How temperature can exist in the presence of essential singularity.

In field theory, there is no barrier to define temperature in the presence of the gluon condensation. Therefore geometric indeterminacy should not mean that we do not have temperature in the presence of condensation. We speculate that the thermalization is incomplete unless the gluon condensation disappears. This is deeply related to the nature of our solution where two independent scales (temperature and gluon condensation) co-exist. Before we answer this question we have even more urgent question. When c is non-zero, the dilaton blows up at the horizon as we already have seen in eq. (2.7). The dilaton blows up at z_f here also. Therefore we should ask:

- Why gluon condensation can exist in spite of the IR divergence of the dilaton?

To answer this question we evaluate the graviton and dilaton action. The matter part is

$$\mathcal{L}_{\text{dilaton}} = -\frac{1}{2}g^{zz}(\partial_z\phi)^2 = -\frac{48c^2z^8}{(1-f^2z^8)^2}, \tag{3.9}$$

which is of course singular. However the gravity part is also singular in this case

$$\mathcal{L}_{\text{grav}} = \mathcal{R} + \frac{12}{R^2} = -8 + \frac{48(f^2 - a^2)z^8}{(1 - f^2z^8)^2}. \tag{3.10}$$

The point here is that the singularity in the dilaton part is canceled by the singularity developed in the gravity-back-reaction to the gluon condensation through the Pythagorean relation $f^2 = a^2 + c^2$. Since the total \mathcal{L} has no IR singularity for any finite values of c , there is no reason why we should not allow it.

The allowance of the non-trivial matter field through the singularity cancelation can be given an interesting interpretation in terms of the boundary condition on the fields: When the IR boundary is a regular horizon, we have to impose a regular boundary condition there. This regularity forbids a non-trivial configuration of matter field ϕ . In the presence of the singularity, classical gravity near singularity is not valid. Therefore the field configuration near the singularity is not reliable. Therefore we have to set an IR cutoff before we reach z_f and the boundary condition should be imposed there. This makes everything regular. In this way, developing singularity is a mechanism by which field configuration can develop a non-trivial value of condensation. In other words, to allow a forbidden quantity by the regularity of the horizon, the system must develop an IR cutoff outside the original horizon. This is the meaning of the singularity located outside original horizon ($f > a$). In fact this is why gluon condensation is allowed in the dilaton-wall solution at zero temperature. We believe that this is a general phenomena of gravity-matter interaction in AdS/CFT.

Now we turn to the question on the temperature. Our answer is that even in the presence of c , we still identify $a = (\pi T)^4/4$. We support this choice by following observation: the temperature in the gravity theory is a parameter that fixes the scale of the geometry and the first non trivial coefficient of in the expansion of the metric, $g_{\mu\nu}^{(4)}$, should determine the scale of the metric. A surprising fact is that even in the presence of c , $g_{\mu\nu}^{(4)}$ does not change:

$$g_{00} = -1 + 3az^4 + \mathcal{O}(z^8), \quad g_{ii} = 1 + az^4 + \mathcal{O}(z^{12}). \tag{3.11}$$

The gluon condensation changes the metric only from the $g_{\mu\nu}^{(8)}$. This can be understood: For the gluons participating in the thermal excitations, they should follow the usual Stephan-Boltzmann law $\rho \sim T^4$, since they are just massless excitations. Therefore, it is natural to identify $g_{\mu\nu}^{(4)}$ as the gluon contribution to the energy momentum tensor $T_{\mu\nu}$ leaving aside the contribution of the glueball or the gluon condensation.

$$g_{\mu\nu}^{(4)} = a(3, 1, 1, 1) = \frac{\kappa^2}{2} \text{diag}(\rho, p, p, p)_{\text{gluon}}. \tag{3.12}$$

This observation supports the identification $a = (\pi T)^4/4$, since the thermal gluons as massless particles should have Stephan-Boltzmann law. However, the identification of the *total* energy momentum tensor with the $g_{ii}^{(4)}$ should be broken to avoid following contradiction: (3.11) indicates that the trace of boundary $T_{\mu\nu}$ is still zero, while we know that the scale invariance is dynamically broken by developing the condensation, introducing one more scale $c = (\pi\Lambda)^4$. Since a and c are independent constants, it is natural to assume that the temperature is independent of the condensation and vice versa. While it is natural to have the notion of in field theoretic temperature in the presence of the gluon condensation, its dual gravity develops an essential singularity and forbid the geometric notion of temperature. The coexistence of two scales and the coexistence of thermal gluon and condensed gluon means that thermalization is incomplete and this, we believe, is the meaning of the singularity in the gravity description.

4. Thermodynamics with gluon condensation

We now discuss the thermodynamic properties of the system. We begin by calculating the total action of gravity and the dilaton:

$$\frac{S_E^{\text{tot}}}{\beta V} = \frac{4}{\kappa^2} \int_{\epsilon}^{z_f} dz \frac{1}{z^5} (1 - f^2 z^8) = -\frac{2}{\kappa^2} f + \frac{1}{\kappa^2 \epsilon^4}, \tag{4.1}$$

where V is the volume of R^3 . We renormalize the total action by subtracting the action value for the pure AdS, which is nothing but the last divergent term in eq. (4.1). To make sure, one can check that this prescription works for the known (AdS black hole) case: It is easy to show that in this case,

$$F/V = -\sigma T^4, \quad E/V = 3\sigma T^3, \quad p = \sigma T^4, \tag{4.2}$$

with $\sigma = \frac{\pi^4}{2\kappa^2} = \frac{\pi^2 N^2}{8}$, so that we can recover well-known Stephan-Boltzmann law with no condensation case $\rho = \frac{3}{8}\pi^2 N^2 T^4$.

In our case,

$$F/V = -\alpha f(T), \quad \rho = E/V = \alpha(-f + T f'(T)), \quad p = \alpha f, \tag{4.3}$$

with $\alpha = \frac{2}{\kappa^2} = \frac{N^2}{2\pi^2}$. With the parametrization $c = \frac{1}{4}(\pi\Lambda)^4$ and the temperature identification

$$a = \frac{1}{4}(\pi T)^4 \tag{4.4}$$

made in last section, the energy density is given by

$$\rho(T, \Lambda) = \alpha(3a^2 - c^2)/f = \frac{3\pi^2 N_c^2 (T^8 - \frac{1}{3}\Lambda^8)}{8\sqrt{T^8 + \Lambda^8}}. \tag{4.5}$$

The result is only for high temperature, since for low temperature other back ground is dominating. If we identify the dilaton (or gluon condensation) contribution of the energy momentum tensor as the difference of total and the thermal gluon identified before:

$$T_{\mu\nu}^{\text{GC}} = T_{\mu\nu}^{\text{total}} - T_{\mu\nu}^{\text{gluon}} = \alpha \text{diag}(Tf'(T) - f - 3a, f - a, f - a, f - a). \tag{4.6}$$

Notice that the gluon condensation contributes negative energy:

$$\rho_{\text{GC}} = -\alpha(3a(f - a) + c^2)/f < 0, \tag{4.7}$$

which is a reminiscent of the zero temperature result of Shifman, Vainstein and Zakharov [21]. In both case, the negativeness is coming from the renormalization.

At $c = 0$ it goes to the usual Stephan-Boltzmann law. The pressure can also be calculated to be

$$p = \alpha f = \frac{\pi^2 N_c^2}{8} \sqrt{T^8 + \Lambda^8} \tag{4.8}$$

so that the trace anomaly due to the gluon condensation is

$$\rho - 3p = -4\alpha \frac{c^2}{f} < 0. \tag{4.9}$$

For large temperature it goes like $\sim -1/T^4$.

In the absence of the proper horizon, the most interesting thermodynamic quantity is the entropy. The entropy density is

$$s = \frac{4\alpha a}{T} \cdot \frac{a}{f} = \frac{\pi^2}{2} N_c^2 T^3 \cdot \frac{T^4}{\sqrt{T^8 + \Lambda^8}}. \tag{4.10}$$

Notice that the entropy is decreased compared with the case with no gluon condensation by the factor a/f . This make sense, since the entropy in condensed state should be less than that in thermal state. In high temperature limit, all the thermodynamic quantities saturate to the case of $c = 0$, regardless of c .

5. Hawking-Page transition

So far we have discussed high temperature regime. Here we discuss the low temperature regime and the phase transition by discussing the deconfinement phase transition along the line of [27, 28]. We first evaluate the value of the action with the ‘thermal dilatonic AdS’ background (tdAdS), which can be obtained by the double Wick rotation (Wick rotation and compactifying the time) of the dilaton-wall solution. We take $R = 1$ here and hereafter. Using the result for curvature of the tdAdS,

$$\mathcal{R}_{\text{tdAdS}} = -4 \cdot (5c_0^4 z^{16} - 22c_0^4 z^8 + 5)/(1 - c_0^2 z^8)^2, \tag{5.1}$$

the action is given by

$$S_E^{\text{tdAdS}}/V_3 = -\frac{4}{\kappa^2} \int_0^{\beta'} dt \int_\epsilon^{z_c} dz \frac{1}{z^5} (c_0^2 z^8 - 1), \quad (5.2)$$

where $z_c = c_0^{-1/4}$ and $V_3 = \int d\vec{x}$ is the volume of our three dimensional space. The action for the dilatonic black hole (dBH) solution is given by

$$\frac{S_E^{\text{dBH}}}{V_3} = -\frac{4}{\kappa^2} \int_0^\beta dt \int_\epsilon^{z_f} dz \frac{1}{z^5} (f^2 z^8 - 1), \quad (5.3)$$

where $z_f = f^{-1/4}$. We used the fact that the curvature in this back ground is

$$\mathcal{R}_{\text{dBH}} = -\frac{4 [5 - 22f^2 z^8 + 5f^4 z^{16} + 12a^2 z^8]}{(1 - f^2 z^8)^2}. \quad (5.4)$$

We determine β' of tdAdS in terms of β by requiring that the periods of the two backgrounds in the compactified time direction are the same at $z = \epsilon$:

$$\beta' = \beta \exp[A(\epsilon) + B(\epsilon) - A_0(\epsilon)] \simeq \beta \left(1 - \frac{3}{2} a \epsilon^4\right), \quad (5.5)$$

where $A_0(\epsilon) = A(\epsilon; a = 0)$. We calculate $\Delta S := S_E^{\text{dBH}} - S_E^{\text{tdAdS}}$:

$$\Delta S = \frac{2}{\kappa^2} \left[c_0 + \frac{3}{4} a - f \right] \beta V_3. \quad (5.6)$$

Hawking-Page transition is at

$$a_{\text{crit}} = \frac{12}{7} c_0 \left(1 + \frac{1}{3} \sqrt{16 - 7(c/c_0)^2}\right), \quad (5.7)$$

and condition for the real solution is $c < 1.51c_0$, but since we expect $c < c_0$, there is always a phase transition. When $c \ll c_0$, we get

$$T_c \simeq \sqrt{2}\Lambda, \quad \text{with } c_0 = (\pi\Lambda)^4/4. \quad (5.8)$$

We see that the scale of the gluon condensation provide a cut-off scale. In hard wall model analyzed in the previous section, $1/z_m$ plays the same role. Due to the first order nature of the phase transition, we could not predict the value of c . The condensation is piecewise constant function of T , which has a drop at a certain temperature T_c , which we can estimate below.

The general structure of temperature dependence of our metric with the Hawking-Page transition is qualitatively similar except that at high temperature $c = 0$ for hard wall model while $c \neq 0$ with gravity back-reaction.

6. Discussion

In this paper we discussed the back-reaction of the metric with the gluon condensation at the finite temperature. We presented a dilatonic black hole solution interpolating the recently discovered dilaton-wall solution and AdS black hole solution. The result contains two parameters and breaks the conformal invariance. We evaluated the trace anomaly and it is proportional to the gluon condensation. We showed how the system uses the singularity to allow the gluon condensation which is forbidden in the presence of regular horizon.

The solution does not admit the Hawking temperature, unless the condensation parameter vanishes. We discussed how to define the temperature in the presence of the singularity using the holographic correspondence and an extended Stephan-Boltzmann law in the field theory. Although the AdS/CFT deals with strong coupling and the field theory with weak coupling, the structure is the same and we think this structure is not modified as we change the coupling although the coefficient of the each term can be changed as a function of coupling.

In this paper the interpretation of singularity is such that the solution is physical in the region outside a region where singularity is contained. Our view is actually practiced extensively before in the present string community. For example in [14, 40] Constable-Myers solution, which has a essential singularity, is used to discuss the 3+1 dimensional confining gauge theory. However, one might wonder whether one can find singularity free solution by adding some other matter. This is especially relevant because, a singularity often means we are using wrong degree of freedom or missing some degree of freedom. For example, in our case, perhaps one should treat the condensed part of gluon as independent degree of freedom. We hope we can report on this issue near future. While we do not know any example worked out to realize the the senario suggested by referee it is definitely interesting idea worthwhile to be pushed in the future.

We discussed the temperature dependence of the gluon condensation with the Hawking-Page transition(HPT). As in the usual case, there is a jump of the gluon condensation at the critical temperature. Apart from this jump and the determination of the transition temperature, there is not much feature in the temperature dependence. Such character of a physical parameter is shared for all AdS/CFT model with HPT. The large N nature of AdS/CFT forbids to reproduce the details of the temperature dependence found in Lattice results.

We describe some future directions: we did not include a hard wall at z_m since there is already another scale provided by the gluon condensation. It is interesting to discuss the Hawking-Page transition in the presence of three or more independent physical scales. Also it would be interesting to discuss back-reaction of the gravity for other fields like massive scalar as well as vectors. These topics are under progress. It would be also interesting to calculate various physical quantities in the presence of the gluon condensate. We want to come back to this issue in future publications.

A. Regularity of the metric and Hawking temperature

Here we discuss the equivalence of the three facts in the class of our metric:

- (i) Definability of hawking temperature,
- (ii) the regularity of the curvature,
- (iii) the rationality of the metric components.

To see this, note that the relevant part of the dBH metric is

$$ds^2 = -g_{00}dt^2 + \frac{1}{z^2}dz^2, \tag{A.1}$$

where

$$g_{00} = \frac{1}{z^2}(1 - fz^4)^{\frac{1}{2} + \frac{3}{2} \frac{a}{f}}(1 + fz^4)^{\frac{1}{2} - \frac{3}{2} \frac{a}{f}}. \tag{A.2}$$

Near z_f , behavior of the metric can be examined by introducing the coordinate $z = z_f(1 - \rho^\beta)$ and rewriting the metric near $\rho \sim 0$,

$$ds^2 \simeq \beta^2 \left(\rho^{2\beta-2} d\rho^2 + \rho^{\alpha\beta} \frac{F(z_f)}{\beta^2} dt_E^2 \right) \tag{A.3}$$

where $dt_E^2 = -dt^2$ and $F(z) := g_{00}(z)/(1 - \frac{z}{z_c})^{\frac{1}{2} + \frac{3}{2} \frac{a}{f}}$, which can be written as

$$F(z) \equiv \frac{1}{z^2} (1 + z^4/z_f^4)^{\frac{1}{2} - \frac{3}{2} \frac{a}{f}} (1 + z/z_f + z^2/z_f^2 + z^3/z_f^3)^{\frac{1}{2} + \frac{3}{2} \frac{a}{f}}. \tag{A.4}$$

The standard lore to get the temperature is to request the absence of a conical singularity in eq. A.3. However, this condition is met only when $\beta = 1$ and $\alpha = 2$, which means $f = a$. This in turn gives $c = 0$. Only for this case we have the well known result for the black hole temperature: $T = \sqrt{2}/\pi z_f$. The Riemann curvature scalar is finite only if $f = a$ and also $f = a$ gives the rational metric components as is manifest in the AdS Schwarzschild solution. One should better watch whether these equivalences are more general phenomena.

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